One of the most fundamental tenets of financial management relates to the time value of money. The old adage that a dollar in hand today is worth more than a dollar in hand tomorrow, or next week, or next month, or next year, etc. provides an illustration of this concept.

The question remains as to why monetary resources are more valuable from a financial sense in the present as opposed to the future. The concept of **opportunity cost** is relevant to the explanation of the time value of money principle. Very simply, money in hand today can be utilized towards its most useful (and productive) purpose, whereas money deferred to the future presumably cannot.

As was stated in an earlier discussion of financial management principles, the financial value assigned to any productive asset (whether real or financial) is estimated by the present value associated with all future cash flows that the asset is projected to produce over its useful life.

As examples, consider two assets -- real estate (real asset) and a share of stock for General Electric (financial asset). The value of each asset at a particular point in time is based on the present value of expected future cash flows generated by each type of asset. In the case of the real asset (real estate), the expected future flow(s) of cash are derived from the expected appreciation in the value of the asset over time and any capital gains derived from the sale of the asset. In the case of the financial asset (GE stock), the expected future flows are derived from the expected future earnings of the company, which should translate into either/both dividends and/or capital gains based on those earnings.

In either case, it is the expectation of future cash flows that determines the present value of each asset. In general, as expected future cash flows increase, the present value of any asset will also increase; conversely, as expected future cash flows decrease, the present value of any asset will also decrease.

It thus follows from this discussion that one of the most basic (and fundamental) tools of financial analysis involves the analysis of cash flows derived from the use (potential use) of various types of assets over time. Within such forms of analysis, there are two primary subsets of cash flow analysis that are most relevant -- future value (compounding) forms of analysis and present value (discounting) forms of analysis.

While both forms of analysis are important and relevant, it is primarily the analysis of present value of future cash flows (discounted cash flow analysis) that forms the basis for most asset valuation problems in financial management.
** Common Terminology

** Present value -- estimate of current value of future cash flows derived from the use of an asset (PV)

** Future value -- estimate of value of future cash flows derived from the use of an asset at some point in the future (FV)

** Interest rate -- opportunity cost or "rent" associated with deferring the use/receipt of scarce monetary resources into the future. (compounding) (I or I/Yr)

** Discount rate -- mathematical inverse of an interest rate, represents the rate at which future cash flows are "discounted" to account for time preference (present vs. future)

** Cash flow -- receipt of (cash inflow) or disbursement of (cash outflow) monetary resources related to the use of a particular asset (CF_j)

** Time line -- visual representation of the time horizon within which all relevant asset cash flows are presumed to occur (N).

** Lump sum -- one time cash flow related to the use of a particular asset

** Annuity -- periodic cash flows of equivalent size/amount associated with the use of a particular asset (PMT)

** Uneven cash flows -- non-annuitized or otherwise irregular cash flows associated with the use of a particular asset

** Future Value Analysis

** Used to estimate the value of some projected stream of revenues/cash flows at some point in the future. Such analyses make use of the financial principle of compounding, whereby future values of asset flows are increased by some multiple of the stated rate of interest.

** For the time being, the assumption is made that compounding of interest occurs only once per year. Later, this assumption will be modified to allow for multiple times per year compounding of interest.

** Future value of lump sum (e.g. invest $1000 in bond paying 8% interest; what is the future value of that investment after 5 years?):

\[ FV_n = PV \times (1 + I)^n \]

** Where 'I' represents the stated rate of interest for purposes of compounding and 'n' represents the time period in the future (in years) to which future value is estimated.

** Given information on the present value of the lump sum cash flow (usually a cash outflow in this case at time 0), the stated rate of interest, and the time period (n) of interest, the future value at time can be calculated w/ a scientific calculator, a financial calculator or a spreadsheet.
** Future value of an annuity (e.g. put $1000 a year in a money market account paying 5% interest per year what is the future value of that account after 5 years?)**

** As mentioned above, an annuity involves equivalent cash flows that occur at regular intervals, an example of which was provided above. Such cash flows may occur daily, weekly, monthly, quarterly, or yearly; so long as the cash flows are the same for each period, future value estimates are based on the annuitized stream of flows over time.

** There are two primary classes of annuities: ordinary annuities, whereby all cash flows are presumed to occur at the END of each time period, and annuities due, whereby all cash flows are presumed to occur at the BEGINNING of each time period.

** In terms of calculation, the estimate of future value of an annuity stream is based on the sum of 'x' lump sum payments compounded forward to time 'n' as follows:

$$ FV_{ordinary\_annuity} = \sum PV_j \times (1 + i)^{n-j} $$

$$ FV_{annuity\_due} = \sum PV_j \times (1 + i)^{n-1-j} \times (1 + i) $$

** Where PV_j represents the value of the annuity cash flow occurring during period j, 'i' represents the stated interest rate for purposes of compounding, and 'n' represents the time period over which the annuitized stream of cash flows is compounded.

** Present Value Analysis

** Used to estimate the present value of a projected stream of cash flows over time associated with the use of a particular asset. As stated before, such present value estimates form the basis for the valuation of many types of assets, both real and financial.

$$ PV = \frac{FV_n}{(1+i)^0} $$  (for lump sum cash flows)

** In this example, the present value calculation is simply a mathematical derivation of the future value equation presented before. The rate of interest (i) is this case is referred to as the discount rate, which enumerates the opportunity cost of a specific use of monetary resources as measured by the value of the next best alternative use of those resources at a given point in time.
Operationally, the discount rate is typically chosen to reflect the rate of interest/return that could reasonably be obtained on investments/assets of similar risk. As it pertains to the choice of discount rate, it is important to note that the specific source of funds/monetary resources is not relevant. In effect, the cost associated with various sources of capital/monetary resources (e.g. debt, cash, equity) is not/should not be a factor in the determination of an appropriate discount rate. This rate should singularly reflect the expected rate of return/interest that could be earned with those funds, regardless of source, on an investment of similar risk.

The estimation of the present value of an annuitized stream of cash flows utilizes the same approach as the lump sum example from above. In this case, as shown below:

$$PV_{\text{ordinary annuity}} = \sum FV_j (1+i)^j$$

$$PV_{\text{annuity due}} = PV_{\text{ordinary annuity}} (1+i)$$

Where $FV_j$ represents the annuity cash flow during period $j$, '$i$' represents the stated discount rate for this stream of annuitized flows, and '$j$' represents the period (year) in which the cash flow was realized.

The present value of a perpetuity, which is defined as an annuitized stream of cash flows that is expected to be realized over an indefinite period of time, is calculated as follows:

$$PV_{\text{perpetuity}} = \frac{\text{Payment}}{\text{Interest (discount rate)}}$$

As can be seen from the above PV examples, the present value of any future cash flows, whether lump sum or annuitized, depends heavily upon the applicable discount rate applied; which, again, is based on the return or interest that could be earned on the next best investment alternative of similar risk. Thus, it can be inferred that present value and interest rates are inversely related to one another (bond example).

** Calculation of Interest Rate and Time Period

If we know/are given the estimates for present value, future value, and time period, it is fairly straightforward to estimate the implied interest rate associated with a stream of cash flows (either lump sum or annuitized) by solving for the interest rate variables in either/both the FV/PV equations.

Alternatively, if we know/are given estimates for present value, future value, and interest rate, it is straightforward to estimate the implied period of time ($N$) necessary for a stream of cash flows to reach FV given PV and stated interest rate. (rule of 72 as an example)
** Time Value Analysis of Uneven Cash Flows

** Time value analysis of uneven cash flows is synonymous with the analysis of a series of unequal lump sum cash flows occurring at different intervals over time (CFj).

** In terms of common applications, both future value and present value estimates can be calculated if data are provided on the nature (amount) and timing of all cash flows and the stated rate of interest/discount rate are provided as well.

** Unlike the case of annuitized cash flows, however, calculation by various means is more involved with uneven cash flows, as each unique cash flow must be entered separately (as one would do with a series of lump sum flows) and either compounded or discounted by the appropriate factor of the interest or discount rate.

** The most commonly utilized application involving the time value analysis of uneven cash flows over time is net present value analysis associated with the evaluation of alternative investment opportunities (more later). At this point, one could easily calculate an NPV for a given investment as follows:

\[ \text{NPV} = \text{Sum of discounted cash flows from investment} - \text{Initial investment} \]

\[ \text{NPV} = \frac{FV_n}{(1+i)^n} - \text{Initial investment} \]

** Another fairly common application that is related to NPV analysis is the estimation of IRR (internal rate of return) for a given investment alternative. In this case, IRR represents the implied rate of return/interest associated with an investment that equates the present value of all investment cash flows with the initial investment. (where NPV = 0) (more on this later as well)

** Miscellaneous Issues in Time Value Analysis

** Alternative compounding periods (other than once a year) are possible (and fairly common) as they relate to the estimation of future value of a stream of cash flows. Examples include daily, weekly, monthly, quarterly, and semi-annual compounding periods.

** When there is more than one compounding period in a given year, more than one rate of interest is plausible for a given set of cash flows:
** Stated rate of interest -- list/coupon rate (annualized)
** Effective annualized rate of interest (EAR) -- real (annualized) rate of return on investment where compounding occurs more than once per year.

\[
\text{EAR} = \left(1 + \frac{\text{stated rate of interest}}{M}\right)^M - 1
\]

** Where M represents the number of compounding periods per year and the stated rate of interest (annual) is provided. For example, a 5% rate of interest with quarterly compounding (4x per year) results in an EAR of 5.095%.

** Loan Amortization -- another common application of time value analysis where annuitized cash flows (payments) are estimated for a given loan amount (present value of loan proceeds) and rate of interest. The calculation of PMT (periodic payment due) given PV, FV, and stated interest rate allows for the construction of an amortization table, which details all interest and principal payments and remaining principal balances over the life of the loan repayment.